Algorithms in fractions are formulas that have nothing to do with reality:

A critical view according to Gestalt-Dialektik:

a holistic philosophy

of education[1]

Gustavo Vieyra

[1] http://www.gestaltdialektik.com/index.php?area=philosophicum&areaSub=introduction_gestaltdialektik

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Algorithms in fractions are formulas that have nothing to do with reality

In the division of fractions, at least in the way I was taught in Acámbaro, Gto. México, I learned to cross-multiply diagonally as follows:

$$\frac{1}{2} \div \frac{1}{4} = \frac{4}{2} = 2$$

In the USA the numerator and denominator of the second fraction are flipped and then the two are multiplied horizontally:

$$\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1} = \frac{4}{2} = 2$$

In the final analysis we are dealing with a formula in both countries, albeit a formula that says nothing about reality. **The students just get a result, but they do not know its meaning.** However, I could give some credit to the American algorithm whereby in the second fraction (as a divisor) the numerator and denominator are flipped. This would make sense, if and only if, the teacher were to explain to the students the logic behind it, such as multiplying one-fourth by its reciprocal:

$$\frac{1}{4} \times \frac{4}{1}$$

And if one-fourth is multiplied by four-ones, so should one-half as well:

$$\frac{1}{2}\left(\frac{4}{1}\right)$$

In such a case, we would end up having the following scenario:

$$\frac{1}{2}\left(\frac{4}{1}\right) \div \boldsymbol{\frac{1}{4}\left(\frac{4}{1}\right)} =$$

In mathematical terms, we may then be able to transform {***one-fourth times four-ones***} into ***one***:

$$\boldsymbol{\frac{1}{4}\left(\frac{4}{1}\right)} = 1$$

Thus, we have the following scenario:

$$\frac{1}{2}\left(\frac{4}{1}\right) \div 1$$

Furthermore, we should also take into consideration that everything that is divided by itself is equal to itself:

X ÷ 1= X

3 ÷ 1 = 3 **7 ÷ 1 = 7** **49 ÷ 1 = 49** **etc.**

Thus,

$$\frac{1}{2}\left(\frac{4}{1}\right) \div 1 = \frac{1}{2}\left(\frac{4}{1}\right)$$

In essence, we go from:

$$\frac{1}{2} \div \frac{1}{4}$$

into:

$$\frac{1}{2}\left(\frac{4}{1}\right) \div \frac{1}{4}\left(\frac{4}{1}\right)$$

and then into:

$$\frac{1}{2}\left(\frac{4}{1}\right) \div 1$$

Thereby:

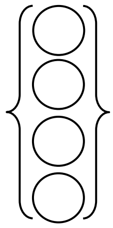
$$\frac{1}{2}\left(\frac{4}{1}\right)$$

and finally into:

$$\frac{1}{2}\left(\frac{4}{1}\right) = \frac{4}{2} = 2$$

Once I asked a university student out of Nicaragua if he could explain to me the result of one {*half-divided by one-fourth*}. He responded by saying that two represented whole numbers, which is completely illogical. If we say that 2 represents whole numbers, then it follows that 2 must represent two real objects as in *two pizzas*. How can we get two pizzas out of {*one-half of a pizza divided by one-fourth of a pizza*}? Likewise, how can we justify, for example, {*one-half by one-eight equals four*} in terms of four objects? Whole numbers are called whole numbers because they must reflect some kind of quantity in real terms as in the following example:

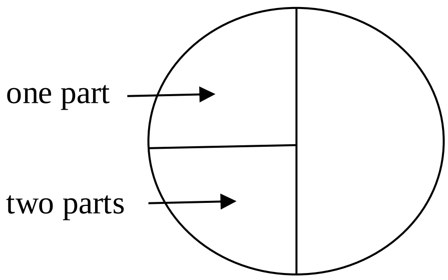
$$\frac{1}{2} \div \frac{1}{8} = \frac{1}{2} \times \frac{8}{1} = \frac{8}{2} = 4$$



Later on, another student explained to me that the answer becomes “undefined” because it has no concrete definition. If the result remains mathematically undefined, we would have to conclude that the 4 remains in the air without having any relation with life itself. This, in my opinion, is ridiculous.

Antithesis: a practical and linguistic reasoning of the fractions

There are people who are very lucid in their mind because they can relate to everything in a very clear and tangible way, even after years of not having studied mathematics. Once I asked a real estate manager if she could give me a logical explanation as to why {*one-half divided by one-fourth equals two*}. Right away she answered that if we were to take one-half of a whole (for example of a pizza) one could further divide that half into two parts:



This holistic interpretation of the fractions is the only one that makes practical sense. In other words, when I say:

$$\frac{1}{2} \div \frac{1}{4}$$

we are dealing with a syntactical conception to the problem, which requires a linguistic interpretation and not a mathematical one. The root of the problem in the division of fractions can best be reflected in terms of language and not in mathematical terms. According to Gestalt-Dialektik (GD), my own holistic philosophy of education[2] (www.gestaltdialektik.com), math is a function of language and not the other way around. In this case, language takes precedence over math because math is based on syntax or syntactical equations and syntax is a part of grammar; syntax plays the main role, the main figure, and not a numerical configuration as portrayed by the classical algorithm in the division of fractions. According to my antithesis, we must reconfigure the problem linguistically and instead of saying:

[2] https://gestaltdialektik.com/index.php?area=philosophicum&areaSub=introduction_gestaltdialektik

https://gestaltdialektik.com/index.php?area=philosophicum&areaSub=dialectical_approach

{*One-half divided by one-fourth*}

we should ask:

{*How often does one-fourth fit into one-half?*}

Thus, it is no longer valid to say {*one-half divided by one-fourth*} because it leads to a fallacy: the 2 would have to be interpreted in terms of whole numbers as in two objects. The only way to interpret {*one-half times four-ones*} (1/2 x 4/1) is by referring to 2 as in *two pizzas* or any *two objects*. There is no other interpretation to {*one-half times four-ones*} and thus 2 as in *two pizzas* cannot be the answer to the original question (*What is one-half divided by one-fourth?*). Under this GD antithesis, we want to find out how many times does **one-fourth** fit into **one-half**. How many times? Two times! Therefore:

$\frac{1}{2} \div \frac{1}{4} =$ 2-t ¼ in ½ (i.e., "twice fits one-fourth in one-half")

Thus, saying for example {*one-half divided by one-fourth*}:

$$\frac{1}{2} \div \frac{1}{4}$$

is linguistically and logically awkward. Instead of looking at the issue in terms of *dividing one-half by one-fourth* we should be asking: *How often does one-fourth fit into one-half?* Therefore:

$\frac{1}{2} \div \frac{1}{4} =$ 2-t ¼ in ½ (i.e., "twice fits one-fourth in one-half")

Another way of interpreting the problem is as follows: *How often does one-half can be divided into fourths?* The answer is *twice* because:

$$\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

Likewise, we could interpret the following problem:

$2 \div \frac{1}{2} =$ 4-t ½ in 2 ("four times fits ½ in 2")

in accordance to the following illustration:

$$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

one two three four "times"

Therefore, 2 ÷ ½ = 4-t ½ (four times ½ in 2) because 4 x ½ = 2 where the 4 becomes the multiplier and not the multiplicand.

Contribution by Stefanie Grotz[3]

A dialogue as dialectical foundation in order to refine the Ansatz (the procedure) of GD

Yes, so it is very good, there you explain what the position of GD is and why. However, one should also clarify that our main concern is to clarify fundamental problems of mathematics. When somebody wants to go deeper into mathematics, then at some point it is clear that {*one-half divided by one-fourth*} is two and one must accept the rule just as it is given in order to be able to process everything in a simple manner. **One cannot reconstruct every operation based on the logic of the addition; that would be very uneconomic.** Nevertheless, it is fundamentally important to understand at the beginning why we do things in certain ways and why the rule functions. In that respect there isn´t enough emphasis in education.

However, I would no longer be able to say that {*one-half divided by one-fourth is two*}. That would be unprofessional because at some point one must be able to reflect a professional language.

Answer from Gustavo Vieyra

A generalization-synthesis according to Gestalt-Dialektik

You write: "One cannot reconstruct every operation based on the logic of the addition; that would be very uneconomic."

That is exactly the point! **One should, in any case, be able to reconstruct every operation based on the logic of the addition**, especially in context of a mental *blitz* in a manner that is highly economic and practical.

The student should be able to experiment with the **"right predicative language"** and with all the corresponding mental demands in order to internalize a new way of thinking. At a certain abstract level, it is fully OK to determine that {*one-half divided by one-fourth equals two*}, but only as generalization of the GD-Postulate:

$$\frac{1}{2} \div \frac{1}{4} = \text{2 times the } \tfrac{1}{4} \text{ into } \tfrac{1}{2} \text{ (thus: "twice fits a fourth into a half")}$$

At the beginning the children must internalize unconditionally the entire **predicative process** based on the logic of the addition. Only then it is fully OK to abstract the 2 through a **mental-predicative *Ansatz***. What does it mean to abstract the 2 in accordance to a mental-predicative *Ansatz*? The students must understand that the 2 does not represent a normal 2. It is, for example, wrong to determine that the 2 in 24 is 2. One makes the addition 9 + 9 + 6 and says, *it is equal to 24*. Up to that point everything is OK. However, when we´re confronted with a more complex addition such as 19 + 29 + 16 then we have syntactic problems. In the USA we may be able to present an addition vertically:

[3] She is a former colleague of GD; a graduate from the University of Passau, Germany in Culture and Economics (KulturWirtschaft) with a Master´s Degree.

$$
\begin{array}{r}
19 \\
+\,29 \\
\underline{16}
\end{array}
$$

In the basic algorithm for additions we say then {*9 + 9 + 6 equals 24; we put the 4 and we carry the 2*} as follows:

$$
\begin{array}{r}
{\scriptstyle (2)} \\
19 \\
29 \\
\underline{16} \\
64
\end{array}
$$

That is precisely the linguistic problem because the 2 in 24 is not a 2, but rather a 20. Linguistically, mathematically and logically speaking it is a fallacy to say {*I put the four and carry the 2*}. It is false and illogical to make such a statement. Thus, I propose that we think differently, thereby internalizing a new "**mental-predicative *Ansatz***" (in terms of a paradigm shift so to speak) in order to eventually be able to internalize something so illogical. In other words, let the student be aware of the illogical principles at hand. In order to get to a higher level of abstraction one must start with the **logic of the addition**, such as in the following example:

> {*Twenty-nine plus nineteen equals twenty-nine plus ten, plus nine; that is equal to forty-eight*}. Numerically: *29 + 19 = [29 + 10 = 39] + 9 = 48*
>
> {*Forty-eight plus sixteen equals forty-eight plus ten, plus six . . .*}
> Numerically: *48 + 16 = [48 + 10 = 58] + 6 = 64*

This long addition is not economic, but it is an effective approach in order to introduce later on the illogical, but faster algorithmic format. If all the factors have been holistically internalized, that is to say, if the students are conscious of the holistic principles at hand, then it would be acceptable to generalize an illogical algorithm, but only as an end-product in context of the **mental-predicative *Ansatz* in accordance to the philosophy of Gestalt-Dialektik.**

In other words, what is it that we need to accomplish in order to make sure that the 2 as an answer to the aforementioned fraction **does not represent a whole number**, **a 2 as in two pizzas**? In order to bring the student to a full and complete understanding of the predicative approach of GD we must introduce fractions in accordance to the logic of the addition as follows:

$\frac{1}{2} \div \frac{1}{4} =$ 2-t the ¼ into ½ (**a predicate of**: "two times fits one fourth in one half")

Making the claim that {*one-half divided by one-fourth equals two*} does not give any clarity in the process. The student gets in theory the right answer; however, the answer is laden with the wrong interpretation because the *two* must be represented as *two objects*, which is completely illogical. One cannot get, for example, two pizzas as the answer to the question, *What is one half of a pizza divided by one-fourth?* That would be like making something out of nothing. It´s impossible to get two pizzas or for that matter any whole units out of the division of fractions. What we get are numbers that function as a **multiplier**, as opposed to a **multiplicand** of a multiplication. In other words, the answer to a division of fraction is an **implied multiplication** in which the first term is a **multiplier**. Thus, when we say 2 as in

$$\frac{1}{2} \div \frac{1}{4} = 2$$

then that *2* is referring to *two-times fits ¼ into ½* in the sense of (2) (¼) = ½. Regardless of how we want to analyse it, the *2* is always functioning as a multiplier. Thus, I propose a mental connotation by introducing the symbol *t* in order to give out a short, quick and precise answer such as for example {*one-half divided by one-fourth equals two-t*} as follows:

$$\frac{1}{2} \div \frac{1}{4} = 2\text{-t}$$

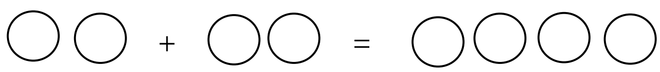
This approach is a logical *connotative-predicative-linguistic-mental solution* to the division of fractions.

When the children understand what the 2 means, in other words, **the 2 as a multiplier** (two times) and not as a multiplicand, then the predicative-connotative short version is a good solution to a fast and quick answer: {*one-half divided by one-fourth is two-t*}.

> **Thesis of Gestalt-Dialektik:**
>
> Most students at the end of high school and at the university level in Germany, the USA and Mexico may or may not be able to comprehend the meaning of 2 in:
>
> {*One-half divided by one-fourth is two*}
>
> If they do, then it is because they learned their arithmetic facts in their elementary school in terms of a broad holistic philosophy of **number sense**. However, if the students learned the fractions and other arithmetic facts via the classic mechanical approach, e.g., by **flipping one-fourth into four-ones and multiplying horizontally**, then most of them would not be able to illustrate with any logic the meaning of the fraction. In others words, if asked to draw some graphic drawings in order to show the procedure as to how they ended up with two as the answer, most students will not be able to make any logical sense of the process and thus, they will get confused with some illogical conclusion. And why is that? That is because they would not have comprehended, holistically speaking, that *2* as the answer to the fraction cannot mean something like *two pizzas*, but rather as a **simple multiplier** in the sense of *twice* as in {*one-fourth fits into one-half twice*}.

According to this thesis we may be able to go to any university, high school and especially to students in the upper elementary school grades and show them how to illustrate for example:

2 + 2 = 4



Having done this, we would then ask the students to make a similar illustration as to the fraction:

$$\frac{1}{2} \div \frac{1}{4}$$

If the students had experienced, in the elementary school years, the facts of arithmetic via a holistic philosophy of number sense, then they may come up with a logical illustration. Otherwise, most will get confused and won´t be able to show any logic behind the numbers, especially those upper elementary school grade students who learned the division of the fractions by flipping the second fraction and multiplying horizontally. The most basic and simple solution is to be practical and give for example the following interpretation:

> Look here is a pizza and if I were to divide it in two then we have two halves. Now, how many fourths fit into one-half? . . . (pause) . . . Two! Thus, *one-half divided by one-fourth is two*, as in *two times the one-fourth fits into the one-half!*

I predict that most university students will even have a lot of trouble explain more complex fractions such for example:

$$\frac{1}{2} \div \frac{1}{3}$$

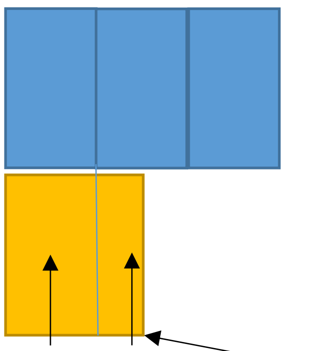
By flipping the 1/3 into 3/1 and multiplying across they may come to the right answer, but nevertheless most university students, especially those in the humanities, will not be able to explain the logic of the answer. They will be fooled by the classic algorithm:

$$\frac{1}{2} \div \frac{1}{3} = \frac{1}{2} \times \frac{3}{1} = \frac{3}{2} = 1\frac{1}{2}$$

Here is the logic according to my GD holistic interpretation:

a) How many times does $\frac{1}{3}$ fit into $\frac{1}{2}$?
b) $\frac{1}{3}$ fits into $\frac{1}{2}$ one and a half times!

$$\frac{1}{2} \div \frac{1}{3} = 1\frac{1}{2} \text{ times fits } \frac{1}{3} \text{ into } \frac{1}{2}$$



$\frac{1}{3}$ plus $\frac{1}{2}$ of $\frac{1}{3}$ fit into $\frac{1}{2}$

In accordance to the "GD predicate Ansatz" we must then write:

$$\frac{1}{2} \times \frac{3}{1} = \frac{3}{2} = 1\frac{1}{2}\,t$$

Orally we would say for t “times” or $1\frac{1}{2}$ *times* fits $\frac{1}{3}$ into $\frac{1}{2}$.

Critical review of Mathe-Stars 6: mechanical and senseless

The division of fractions on pages 36-39 from **Mathe-Stars 6**[4] is mechanical and senseless from a practical point of view. **The algorithmic representation has nothing to do with logic or reality.**

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Division of fractions (1)

1 Reciprocal values

a) For each fraction one can get a reciprocal value.

fraction $\frac{3}{4}$ $\frac{4}{3}$ reciprocal

Form the reciprocal value:

$\frac{2}{3}$ $\quad \frac{3}{\ } \quad \frac{6}{8} \quad \frac{\ }{\ } \quad \frac{9}{5} \quad \frac{\ }{\ } \quad \frac{7}{10} \quad \frac{\ }{\ } \quad \frac{2}{4} \quad \frac{\ }{\ }$

b) Natural numbers may be transformed into their respective reciprocal values.

$5 = \frac{5}{1}$ reciprocal *value* $\frac{1}{5}$

Form the reciprocal value:

8 — 6 — 3 — 2 — 9 —

2 Division of fractions

One divides by a fraction by multiplying by multiplying by the reciprocal.

$$\frac{1}{3} : \frac{2}{5} = \frac{1}{3} \cdot \frac{5}{2} = \frac{1 \cdot 5}{3 \cdot 2} = \frac{5}{6} \qquad \frac{1}{4} : \frac{7}{9} =$$

$$\frac{1}{7} : \frac{3}{5} = \frac{1}{7} \cdot \frac{5}{3} = \qquad \frac{1}{6} : \frac{1}{5} =$$

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[4] Mathe-Stars 6 (2015). Werner Hatt, Stefan Kobr, Ursula Kobr, Elisabeth Plankl & Beatrix Pütz (eds.). Berlin: Cornelsen Schulverlag GmbH, www.oldenbourg.de

$\frac{1}{5} : \frac{1}{3} =$

$\frac{1}{2} : \frac{5}{7} =$

$\frac{1}{3} : \frac{4}{7} =$

$\frac{2}{5} : \frac{3}{7} =$

$\frac{1}{5} : \frac{2}{3} =$

$\frac{2}{9} : \frac{5}{6} =$

****

The representation of the division of fractions on pages 36 to 39 is mechanical and thus has nothing to do with the practical reality of a student. It deals with a massive amount of numbers that have no practical meaning for the students at all.

Critique on an X Quadrat MATHEMATIK 6 BADEN-BÜRTTEMBERG: At the beginning is the approach holistic, but at the end relatively mechanical

What about a more holistic perspective in order to make a better representation of the natural numbers? The approach on page 31 in the book *X Quadrat MATHEMATIK 6 BADEN-BÜRTTEMBERG*[5] is excellent at the beginning because of its holistic representation:

****

2 We divide surface areas by portions. Look carefully what happens:

a) How often does the red fit into the green area?

2 wholes are divided into fourths: $2 : \frac{1}{4}$

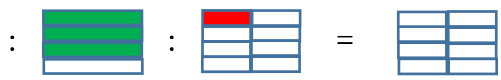
$2 : \frac{1}{4} = 8$

b) How many red fifths fit in 3 green wholes?

3 wholes are divided into fifths.

$3 : \frac{1}{5} =$? ? ?

C) How many red eights fit in 3 fourths of a whole?
3 fourths are divided into eights.



$\frac{3}{4} : \frac{1}{8} =$?

X Quadrat MATHEMATIK 6 BADEN-BÜRTTEMBERG 2017, p. 31

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[5] X Quadrat MATHEMATIK 6 BADEN-BÜRTTEMBERG. Publisher: Dieter Baum & Hannes Klein. Berlin: © 2015 Cornelsen Schulverlage GmbH www.cornelsen.de

During this process the students are able to experience the division of fractions in a real holistic process, beginning with a practical context and with the right linguistic application of the sentence: **how often does a fraction fit into a whole?** Thus, wholes are divided into fourths, fifths, eights in such a way that the logic of the sentence structure becomes crystallized as in: **how often does a fraction fit into a whole?** However, after offering a wonderful holistic and logical perspective the book deteriorates into a senseless mechanization of the division of fractions with the description of the classical algorithm in the form of a math formula on page 31:

Division of fractions

> The division of a fraction can be replaced with the multiplication of the reciprocal value.
> One is able to get the **reciprocal value** of a fraction by inverting the numerator with the denominator.

There is no holistic explanation as to why the division is solved via the reciprocal value. The formula is just introduced without any practical application and above all, without any connection to reality in terms of **number sense** or a real-world representation. On page 32 an algorithmic exercise is presented by asking the students to **determine** the reciprocal value and by **solving** certain fractions:

Exercises: 1 Determine the corresponding reciprocal value.

a $\frac{1}{4}$; $\frac{2}{3}$; $\frac{2}{5}$; $\frac{3}{7}$

c $\frac{3}{4}$; $\frac{5}{8}$; $\frac{8}{15}$; $\frac{11}{12}$

b $\frac{27}{41}$; $\frac{35}{12}$; $\frac{171}{100}$; $\frac{45}{92}$

d $\frac{1}{4}$; $\frac{2}{3}$; $\frac{2}{5}$; $\frac{3}{7}$

2 Solve.

a $\frac{3}{7} : \frac{4}{5}$

c $\frac{2}{6} : \frac{5}{9}$

e $\frac{2}{6} : \frac{5}{9}$

b $\frac{1}{4} : \frac{1}{4}$

d $\frac{2}{5} : \frac{3}{4}$

f $\frac{8}{11} : \frac{6}{5}$

X Quadrat MATHEMATIK 6 BADEN-BÜRTTEMBERG, 2015, p. 32

Although a holistic approach is not sufficiently applied a holistic perspective is partially introduced, especially with some very creative and practical questions:

3 How big is portion?

a $\frac{7}{4}$ kg of wood chips is divided into 7 equal parts.

b $\frac{3}{4}$ kg of gypsum is put into 6 bags of the same size.

c $12\frac{1}{2}$ kg of sand are packaged into five equal portions.

X Quadrat MAT.HEMATIK 6 BADEN-BÜRTTEMBERG, 2015, p. 32

To divide a fraction with a whole number is not so difficult because 4 is the same thing as $\frac{4}{1}$.

Also: $\frac{1}{3} : 4 = \frac{1}{3} : \frac{4}{1} = \frac{1}{3} . \frac{1}{4} = \frac{1}{12}$

X Quadrat MAT.HEMATIK 6 BADEN-BÜRTTEMBERG, 2015, p. 32

The approach of this book is not holistic enough because the implied logic of the formula is not explained properly. On the one hand some exercises are very well illustrated and practiced in a holistic, praxis-oriented format and on the other hand the classic algorithm is introduced without any concrete connection to exercises that were practiced holistically. A statement such as:

To divide a fraction with a whole number is not so difficult
because 4 is the same thing as $\frac{4}{1}$.

has no connection to reality; the formula is compelled and forced on the students instead of orienting itself to the praxis and examples given on 3a, 3b and 3c. The teacher should first and foremost invite the students to think and discuss the meaning and implications of these examples:

$$\frac{1}{3} : 4$$

Can one, for example, ask how often does 4 in $\frac{1}{3}$? This question makes no sense; but the teacher should guide the students through dialogue in order to postulate at a holistic perspective between the following two divisions:

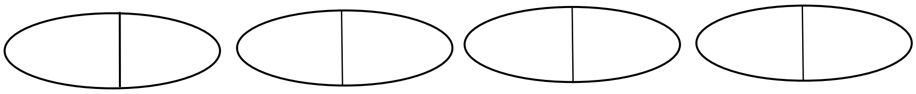
Teacher: What is the difference between $\frac{1}{2} : 4$ and $4 : \frac{1}{2}$?

Student: With $4 : \frac{1}{2}$ one can ask, how often does $\frac{1}{2}$ fit into 4?
but with $\frac{1}{2} : 4$ I have no idea, what should I ask.

Teacher: Can one ask, how often does 4 fit into $\frac{1}{2}$? Yes, one can ask that question, but that make no sense because the 4 is just bigger than $\frac{1}{2}$. A 4 does not fit into $\frac{1}{2}$. With $4 : \frac{1}{2}$ one can logically ask, how often does $\frac{1}{2}$ fit into 4. Eight times as a matter of fact! $\frac{1}{2}$ fits exactly eight times into 4 as follows:

Teacher: Can one ask, how often fits a 4 into $\frac{1}{2}$? Yes, one can ask this question, but it doesn´t make sense, because the 4 is just bigger than $\frac{1}{2}$. A 4

does not fit into $\frac{1}{2}$. However, with $4 : \frac{1}{2}$ one can ask logically, how often does $\frac{1}{2}$ fit in a 4? Eight times as a matter of fact! One $\frac{1}{2}$ fits exactly eight times in 4 as follows (where t = times):



one time : two t   three t : four t   five t : six t   seven t : eight t

One can even depict all of this as a long addition:

$$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

1time : 2 times : 3 times : 4 times : 5 times : 6 times : 7 times : 8 times

At this point the teacher may very well introduce the mechanical, senseless algorithm as a way to find the answer quickly:

Teacher: There is a trick, a formula, in order to solve the divisions of fractions quickly and that is by using the reciprocal value. For example, one can depict 4 as a fraction: $\frac{4}{1}$ and this fraction can be multiplied by the reciprocal value of $\frac{1}{2}$, that is, $\frac{2}{1}$ and voilà, one gets a quick answer:

$$4 : \frac{1}{2} = \frac{4}{1} : \frac{1}{2} = \frac{4}{1} . \frac{2}{1} = \frac{4 \,.\, 2}{1 \,.\, 1} = \frac{8}{1} = 8$$

With $\frac{1}{2} : 4$ one can just divide $\frac{1}{2}$ four times, as in a true division and thus we get four times $\frac{1}{8}$ and therefore $\frac{1}{2} : 4 = \frac{1}{8}$ because:

$$\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{4}{8} = \frac{2}{4} = \frac{1}{2}$$

1 time : 2 times : 3 times : 4 times

Generally, I find the approach on pages 31-33 in the book X Quadrat MATHEMATIK 6 BADEN-BÜRTTEMBERG holistically deficient, but much better than **Mathe-Stars 6**. The book X Quadrat MATHEMATIK 6 should always substantiate the holistic nature from beginning to end so that the students may internalize and incarnate the whole logic of the algorithmic formulas. Therefore, the students may be empowered to understand not just the concept of the reciprocal value, but above all the logic of the algorithmic formula. The formula would then become an ingenious machinery, which one can apply, knowing that it´s artificial without any connection to real life. Students must understand that the formula does not correspond to reality and that it´s only an auxiliary device in order to quickly get to the answer in order to save time. The approach is nevertheless interesting from a holistic perspective, especially sections 2a, 2b and 2c on page 31 as well as sections 3a, 3b and 3c on page 32 with a follow up on p. 33.

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